Limits → algebraic evaluation

→ L'Hospital's Rule

→ subtleties regarding zeros in denominator

→ horizontal/vertical asymptotes as limits

→ AND...

Limit Def^n of Derivative

idea: derivative = limit (slope of secant lines)

4 forms:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

differentials

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f'(x) = \lim_{w \to x} \frac{f(w) - f(x)}{w - x}$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Answer on AP $\Rightarrow$ take the derivative
plug in point

Derivatives

→ instantaneous rate of chg

→ slope of tangent line

[[given a point (a,b) and $f'(a)=m$
eqn of tangent is
$y-b=m(x-a)$]]

slope is a NUMBER

→ Approximations

* approximate instantaneous rate of change with average rate of change

* approximate value of function with value of tangent line "local linear approximation"

$$f(x) \approx f(a)+f'(a)(x-a)$$

→ related rates (of change)

purpose - find at a point

treat $x, y, z, a, b, m, r, g, w, \ldots$ as fn of time (t)
derivative with respect to t

$y = \begin{cases} x^2, & x>0 \\ x^2, & x\le 0 \end{cases}$

→ optimization

purpose - find max or min

- one thing chgs wrt another
- take derivative wrt
- problem
  - find function
  - express it in terms of 1 variable

cont
$\lim_{x\to a^+} y = \lim_{x\to a^-} y$

diff
$\lim_{x\to a^+} y' = \lim_{x\to a^-} y'$

Derivatives & Curve sketching

$f'(x)$

- increasing vs decreasing

$f'(x) > 0$ $f'(x) < 0$

max: $f'(x) > 0$

f^n: increasing $\to 0^*$ $\to$ decreasing

$f'(x) < 0$

* max may occur when f' undefined

min: f^n: decr $\to 0 \to$ inc

min: $f'(x) < 0 \Rightarrow \underset{\text{und}}{0} \Rightarrow f'(x) > 0$

Absolute max/min

→ abs max is the highest y-value that occur

→ must be a rel. maximum

OR

an endpoint.

$f''(x)$ - Concavity

$f'(x) = g(x)$

$g'(x) > 0 \Rightarrow$
$g(x)$ increasing

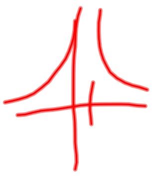
Concave up $\equiv f''(x) > 0$
$\equiv f'(x)$ INCREASING

Concave down $\equiv f''(x) < 0$
$\equiv f'(x)$ decreasing

- point of inflection
Pt at which concavity changes

Either Concave up $\rightarrow$? $\rightarrow$ concave dn
$0 = f''$
Und $= f''$ [[max of first derv]]
$\rightarrow f'$ inc $\rightarrow$ 0/Und $\rightarrow f'$ dec

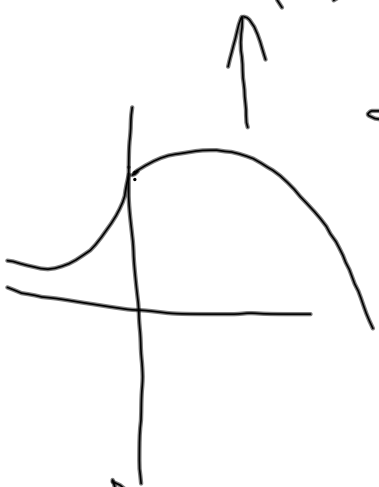
OR concave dn $\rightarrow$? $\rightarrow$ concave up
$\rightarrow f'$ dec $\rightarrow$ 0/Und $\rightarrow f'$ inc



POI?

pt of inflection is a pt at which concavity chgs.

not a pt on curve



POI is considered to be a property of a continuous curve

POI

Alternating Series

- not just a series where + and − are randomly assigned
- but a series where signs actually alternate

Q for A.S.

Do you converge?

Ans:

If $|a_1| \geq |a_2| \geq |a_3| \geq |a_4| \cdots$

AND

$\lim_{n\to\infty} a_n = 0$

THEN series converges.

p691 ⇒ summary of all convergence tests.

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+-\cdots-$$

Alternating Harmonic

Converges (to $\ln 2$)

Convergence

- Absolute Convergence

A series $\sum_{k=1}^{\infty} a_k$ converges absolutly if $\sum_{k=1}^{\infty} |a_k|$ converges

- Conditional Convergence

A series conv. conditionally if it converges but doesn't conv. absolutly.

How do I show abs convergence of an A.S.

big test: ratio test

$$L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

if $L > 1$ no

$L < 1$ yes

$L = 1$ idk

1 interesting Quirk

→ A rearranged absolutely convergent series still converges to the same thing, as the unarranged abs. conv. series did.

→ A rearranged conditionally convergent series could converge to ANYTHING (and it's worse...)

1 practical nicety

HW/ 10.7/ 1-23 odd due 3/26

If alternating series has decreasing $|a_k|$ and $\lim_{k\to\infty} a_k = 0$

THEN

$|S - S_n| < |a_{n+1}|$

$\left|\left(1 - \frac{1}{2}\right) - \ln 2\right| < \frac{1}{3}$

$\left|\left(1 - \frac{1}{2} + \frac{1}{3}\right) - \ln 2\right| < \frac{1}{4}$

$\vdots$